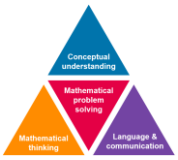
Conceptual Understanding Framework



1. Mathematical concepts can be represented in many ways.

Representations can be concrete, pictorial / diagrammatic, symbolic or linguistic and one way learners construct meaning is by moving between different representations of the same concept. Different representations stress and ignore different aspects of a concept and so making explicit links between a range of representations allows the learner to construct a robust and comprehensive conceptual framework. For example, the array can draw attention to the commutative and distributive properties of multiplication, while equal jumps on a number line emphasise multiplication as repeated addition. The learner needs to recognise both as equally valid models of multiplication, to be drawn upon in different contexts.

How would you answer these questions?

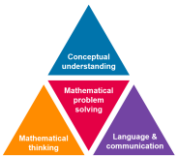
- What different representations (concrete, pictorial / diagrammatic, symbolic, linguistic) relate to the concept?
- What does each representation of the concept stress and ignore?
- What questions and tasks might you offer to learners to help them to make connections between different representations of the same concept?
- What are the key features, misconceptions (or intuitive understandings / naïve beliefs) and difficulty points of the concept?
- Which representations will be useful in highlighting the key features, misconceptions and difficulty points of the concept?
- What might you do / say / write / keep the same / vary / emphasise to direct learners' attention to the non-arbitrary and substantive features of the concept?

2. Learners must create connections between concepts. Representations can help them to make these connections.

Mathematics is an interconnected subject and if learners are to develop a conceptual understanding, rather than memorising a long list of procedures, they need to make sense of the connections between concepts. One view of mathematical understanding is as a network of interconnected concepts; the better one's ability to reason about the connections between the concepts, the better the understanding. Purposeful choice of representations that emphasise the underlying structure can increase the likelihood of learners making sense of the connections between concepts that share underlying structures. It is therefore important to have a clear rationale for the use of a particular representation to teach a specific concept, and to draw attention to the connections between representations and concepts. However, only the learner can construct their own multiple connections; the teacher cannot 'transmit' the connections.

How would you answer these questions?

- What are the connections between this concept and other concepts?
- What are the connections between different representations within the concept?
- Which representations best draw attention to those connections?



- What kinds of experiences with those representations will learners need to have for it to be likely they will see the connections you have identified?
- What opportunities will you provide for learners to move between representations and to create their own representations?
- How does the concept you are teaching now develop and / or link to other concepts learners will encounter in the future? How do your choices now prepare learners for this?

3. Learners must activate prior understanding to connect it to new ideas. Representations can support this.

Learners are not 'empty vessels' or 'blank slates'. They enter our classrooms with different pre-existing conceptual frameworks, based on their individual prior experiences. These may be partial or naïve conceptions, or intuitive understandings. For example, if a learner has only experienced multiplication with the natural numbers, their generalisation that 'multiplication makes bigger' is justifiable, based on prior experience.

Learners must activate their pre-existing understanding to assimilate or accommodate new experiences into these frameworks. Familiar representations can provide a bridge from prior learning to new, more complex understandings, including those that present a cognitive conflict. For example, if a learner is confident with the meaning of the area model of multiplication, this can help them to understand how multiplication by numbers between 0 and 1 results in a product smaller than the multiplicand. It is therefore important to find out about learners' existing conceptual frameworks and the representations with which they are familiar, in order to provide experiences that enable learners to reorganise and expand their current understanding, rather than layer new, disconnected ideas on top of old ones.

How would you answer these questions?

- How can you find out learners' current conceptual frameworks? What questions or tasks might you use?
- Which representations of the concept are learners familiar with? Which are new? How will you support learners in making connections between the familiar and the new?
- Do any representations have the potential to surprise learners by contradicting or conflicting with their current concept image?
- How will you use this information to plan a learning sequence that offers a coherent conceptual development that makes sense to learners?
- How does the concept you are teaching now develop and / or link to other concepts learners will encounter in the future? How do your choices now prepare learners for this?

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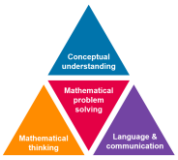

4. Teacher example and task choice is important in creating a conceptual narrative.

The examples and tasks teachers choose and the order in which they are sequenced can influence the development of learners' mathematical conceptions. By juxtaposing specific representations and sequencing them purposefully, whereby some aspects are kept the same while others are varied (called variation), it is possible to increase the likelihood that the underlying relationships become discoverable to learners.

How would you answer these questions?

- What are typical examples of the concept? Unusual examples? Non-examples?
- When planning a sequence of representations or examples, what might be varied and what might be kept the same? What might this help learners to see?
- What is the purpose of the task you have chosen? Does the task fulfil this purpose?
- What are the possible learner responses to the task, both correct and incorrect? How will you respond to these in turn?
- What questions and prompts might you use while learners work on the task(s) to draw their attention to key features of the concept and address misconceptions?

5. Learners must ultimately be able to use the representations as independent tools for thinking, rather than becoming reliant on a specific representation to solve a 'type' of problem.

Representations, including manipulatives, are important tools for thinking and reasoning. However, learners can become reliant on specific representations or manipulatives to solve specific problem 'types', for example, needing to use base ten equipment to calculate the answer to an addition. In the example of base ten equipment, the manipulative allows the learner to engage with the structure of number, which would be hidden in written form. As the learner interacts with the representation, they develop a mental model of this structure that is generalisable to a range of situations. The representation therefore becomes a tool for thinking. Some representations are particularly versatile in their potential to connect concepts, for example, the number line, the array, the bar model. These can become particularly powerful tools for thinking but they will only become so if used regularly enough to be internalised by learners.

How would you answer these questions?

- Which representations are your learners able to use as tools for thinking? To what extent? How do you know?
- Are any learners dependent on particular representations to solve specific types of problems? How can you support them to move towards using the representation as a tool for thinking?